14-1 Counting Outcomes

What You'll Learn

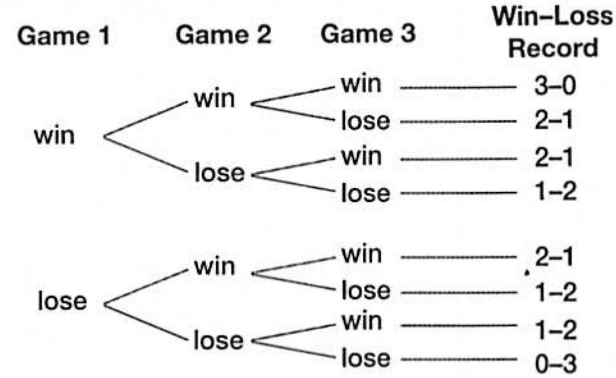
- Count outcomes using a tree diagram.
- Count outcomes using the Fundamental Counting Principle.

Vocabulary

- tree diagram
- sample space
- event
- Fundamental Counting Principle
- factorial

How are possible win–loss records counted in football?

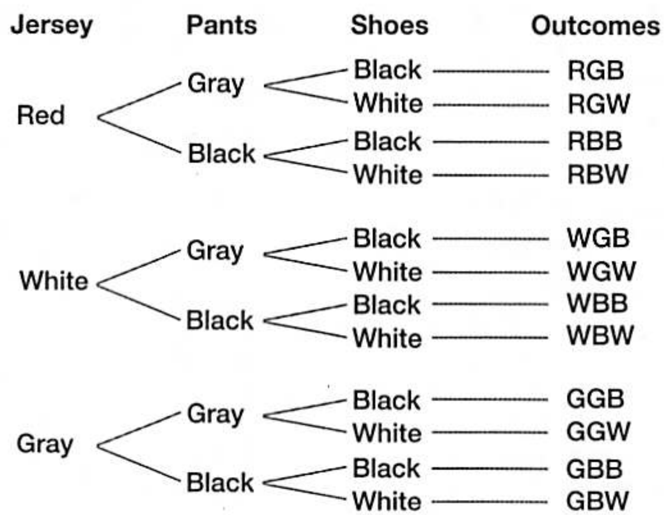
The championship in the Atlantic Coast Conference is decided by the number of conference wins. If there is a tie in conference wins, then the team with more nonconference wins is champion. If Florida State plays 3 nonconference games, the diagram at the right shows the different records they could have for those games.



TREE DIAGRAMS One method used for counting the number of possible outcomes is to draw a **tree diagram**. The last column of a tree diagram shows all of the possible outcomes. The list of all possible outcomes is called the **sample space**, while any collection of one or more outcomes in the sample space is called an **event**.

Example 1 Tree Diagram

A football team uses red jerseys for road games, white jerseys for home games, and gray jerseys for practice games. The team uses gray or black pants, and black or white shoes. Use a tree diagram to determine the number of possible uniforms.



The tree diagram shows that there are 12 possible uniforms.

THE FUNDAMENTAL COUNTING PRINCIPLE The number of possible uniforms in Example 1 can also be found by multiplying the number of choices for each item. If the team can choose from 3 different colored jerseys, 2 different colored pants, and 2 different colored pairs of shoes, there are $3 \cdot 2 \cdot 2$ or 12 possible uniforms. This example illustrates the **Fundamental Counting Principle**.

Key Concept — Fundamental Counting Principle

If an event M can occur in m ways and is followed by an event N that can occur in n ways, then the event M followed by event N can occur in $m \cdot n$ ways.

Example 2 Fundamental Counting Principle

The Uptown Deli offers a lunch special in which you can choose a sandwich, a side dish, and a beverage. If there are 10 different sandwiches, 12 different side dishes, and 7 different beverages from which to choose, how many different lunch specials can you order?

Multiply to find the number of lunch specials.

$$\underbrace{10}_{\text{sandwich choices}} \cdot \underbrace{12}_{\text{side dish choices}} \cdot \underbrace{7}_{\text{beverage choices}} = \underbrace{840}_{\text{number of specials}}$$

The number of different lunch specials is 840.

Example 3 Counting Arrangements

Mackenzie is setting up a display of the ten most popular video games from the previous week. If she places the games side-by-side on a shelf, in how many different ways can she arrange them?

The number of ways to arrange the games can be found by multiplying the number of choices for each position.

- Mackenzie has ten games from which to choose for the first position.
- After choosing a game for the first position, there are nine games left from which to choose for the second position.
- There are now eight choices for the third position.
- This process continues until there is only one choice left for the last position.

Let n represent the number of arrangements.

$n = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or 3,628,800

There are 3,628,800 different ways to arrange the video games.

The expression $n = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ used in Example 3 can be written as 10! using a **factorial**.

Key Concept — Factorial

- **Words** The expression $n!$, read n factorial, where n is greater than zero, is the product of all positive integers beginning with n and counting backward to 1.
- **Symbols** $n! = n \cdot (n - 1) \cdot (n - 2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$
- **Example** $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or 120

By definition, $0! = 1$.

Example 4 Factorial

Find the value of each expression.

a. **6!**

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial

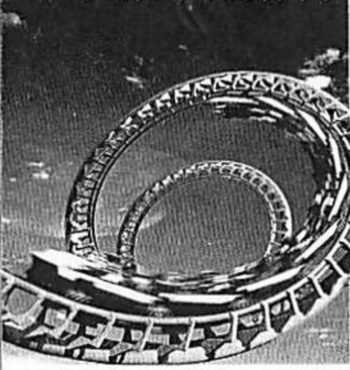
$= 720$ Simplify.

b. **10!**

$10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial

$= 3{,}628{,}800$ Simplify.

More About. . .



Roller Coasters

In 2000, there were 646 roller coasters in the United States.

Type	Number
Wood	118
Steel	445
Inverted	35
Stand Up	10
Suspended	11
Wild Mouse	27

Source: Roller Coaster Database

Example 5 Use Factorials to Solve a Problem

ROLLER COASTERS **Zach and Kurt are going to an amusement park. They cannot decide in which order to ride the 12 roller coasters in the park.**

a. **How many different orders can they ride all of the roller coasters if they ride each once?**

Use a factorial.

$12! = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial

$= 479{,}001{,}600$ Simplify.

There are 479,001,600 ways in which Zach and Kurt can ride all 12 roller coasters.

b. **If they only have time to ride 8 of the roller coasters, how many ways can they do this?**

Use the Fundamental Counting Principle to find the sample space.

$s = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$ Fundamental Counting Principle

$= 19{,}958{,}400$ Simplify.

There are 19,958,400 ways for Zach and Kurt to ride 8 of the roller coasters.

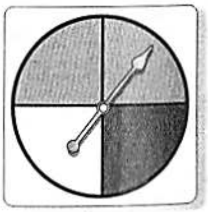
Check for Understanding

Concept Check

1. **OPEN ENDED** Give an example of an event that has $7 \cdot 6$ or 42 outcomes.
2. **Draw** a tree diagram to represent the outcomes of tossing a coin three times.
3. **Explain** what the notation 5! means.

Guided Practice

For Exercises 4–6, suppose the spinner at the right is spun three times.



4. Draw a tree diagram to show the sample space.
5. How many outcomes are possible?
6. How many outcomes involve both green and blue?
7. Find the value of 8!.

Application

8. **SCHOOL** In a science class, each student must choose a lab project from a list of 15, write a paper on one of 6 topics, and give a presentation about one of 8 subjects. How many different ways can students choose to do their assignments?

Practice and Apply

Homework Help

For Exercises	See Examples
9, 10, 19	1
11–14	4
15–18, 20–22	2, 3, 5

Extra Practice

See page 851.

Draw a tree diagram to show the sample space for each event. Determine the number of possible outcomes.

9. earning an A, B, or C in English, Math, and Science classes
10. buying a computer with a choice of a CD-ROM, a CD recorder, or a DVD drive, one of 2 monitors, and either a printer or a scanner

Find the value of each expression.

11. 4!   12. 7!   13. 11!   14. 13!

15. Three dice, one red, one white, and one blue are rolled. How many outcomes are possible?

16. How many outfits are possible if you choose one each of 5 shirts, 3 pairs of pants, 3 pairs of shoes, and 4 jackets?

17. **TRAVEL** Suppose four different airlines fly from Seattle to Denver. Those same four airlines and two others fly from Denver to St. Louis. If there are no direct flights from Seattle to St. Louis, in how many ways can a traveler book a flight from Seattle to St. Louis?

COMMUNICATIONS For Exercises 18 and 19, use the following information.
A new 3-digit area code is needed in a certain area to accommodate new telephone numbers.

18. If the first digit must be odd, the second digit must be a 0 or a 1, and the third digit can be anything, how many area codes are possible?
19. Draw a tree diagram to show the different area codes using 4 or 5 for the first digit, 0 or 1 for the second digit, and 7, 8, or 9 for the third digit.

SOCCER For Exercises 20–22, use the following information.
The Columbus Crew are playing the D.C. United in a best three-out-of-five championship soccer series.

20. What are the possible outcomes of the series?
21. How many outcomes require exactly four games to determine the champion?
22. How many ways can D.C. United win the championship?

23. **CRITICAL THINKING** To get to and from school, Tucker can walk, ride his bike, or get a ride with a friend. Suppose that one week he walked 60% of the time, rode his bike 20% of the time, and rode with his friend 20% of the time. How many outcomes represent this situation? Assume that he returns home the same way that he went to school.

24. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

    How are possible win–loss records counted in football?

    Include the following in your answer:
    - a few sentences describing how a tree diagram can be used to count the wins and losses of a football team, and
    - a demonstration of how to find the number of possible outcomes for a team that plays 4 home games.

Standardized Test Practice

25. Evaluate 9!.

(A) 362,880 (B) 40,320 (C) 36 (D) 8

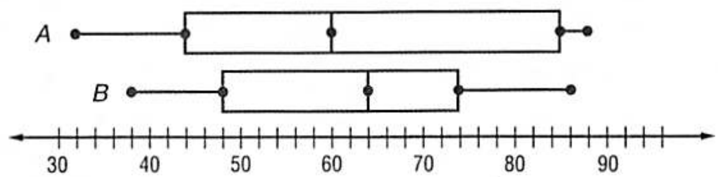
26. A car manufacturer offers a sports car in 4 different models with 6 different option packages. Each model is available in 12 different colors. How many different possibilities are available for this car?

(A) 96 (B) 144 (C) 288 (D) 384

Maintain Your Skills

Mixed Review

For Exercises 27–30, use box-and-whisker plots A and B. *(Lesson 13-5)*



27. Determine the least value, greatest value, lower quartile, upper quartile, and median for each plot.
28. Which set of data contains the least value?
29. Which plot has the smaller interquartile range?
30. Which plot has the greater range?

For Exercises 31–34, use the stem-and-leaf plot. *(Lesson 13-4)*

31. Find the range of the data.
32. What is the median?
33. Determine the upper quartile, lower quartile, and interquartile range of the data.
34. Identify any outliers.

Stem	Leaf
3	0 1 4 5
4	4 4 8
5	6 9
6	6 8
7	1 6
8	0 1
9	
10	9

$3|0 = 30$

Find each sum or difference. *(Lesson 12-7)*

35. $\frac{2x+1}{3x-1} + \frac{x+4}{x-2}$

36. $\frac{4n}{2n+6} + \frac{3}{n+3}$

37. $\frac{3z+2}{3z-6} - \frac{z+2}{z^2-4}$

38. $\frac{m-n}{m+n} - \frac{1}{m^2-n^2}$

Study Tip

Deck of Cards
In this text, a *standard deck of cards* always means a deck of 52 playing cards. There are 4 suits—clubs (black), diamonds (red), hearts (red), and spades (black)—with 13 cards in each suit.

Solve each equation. *(Lesson 11-3)*

39. $5\sqrt{2n^2 - 28} = 20$

40. $\sqrt{5x^2 - 7} = 2x$

41. $\sqrt{x+2} = x - 4$

Solve each equation by completing the square. Round to the nearest tenth if necessary. *(Lesson 10-3)*

42. $b^2 - 6b + 4 = 0$

43. $n^2 + 8n - 5 = 0$

44. $x^2 - 11x - 17 = 0$

45. $2p^2 + 10p + 3 = 0$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **One card is drawn at random from a standard deck of cards. Find each probability.** *(To review **simple probability**, see Lesson 2-6.)*

46. $P(10)$
47. $P(\text{ace})$
48. $P(\text{red } 5)$
49. $P(\text{queen of clubs})$
50. $P(\text{even number})$
51. $P(3 \text{ or king})$