14-3 Probability of Compound Events

What You'll Learn

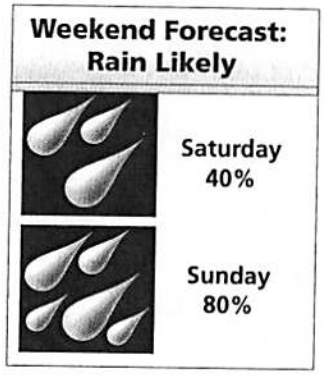
- Find the probability of two independent events or dependent events.
- Find the probability of two mutually exclusive or inclusive events.

Vocabulary

- simple event
- compound event
- independent events
- dependent events
- complements
- mutually exclusive
- inclusive

How are probabilities used by meteorologists?

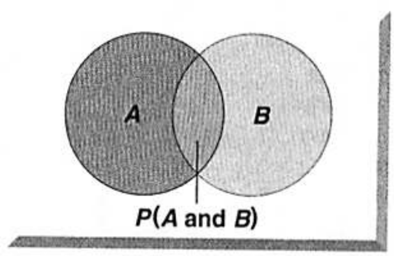
The weather forecast for the weekend calls for rain. By using the probabilities for both days, we can find other probabilities for the weekend. What is the probability that it will rain on both days? only on Saturday? Saturday or Sunday?



INDEPENDENT AND DEPENDENT EVENTS A single event, like rain on Saturday, is called a **simple event**. Suppose you wanted to determine the probability that it will rain both Saturday and Sunday. This is an example of a **compound event**, which is made up of two or more simple events. The weather on Saturday does not affect the weather on Sunday. These two events are called **independent events** because the outcome of one event does not affect the outcome of the other.

Key Concept — *Probability of Independent Events*

- **Words** If two events, A and B, are independent, then the probability of both events occurring is the product of the probability of A and the probability of B.
- **Symbols** $P(A \text{ and } B) = P(A) \cdot P(B)$
- **Model**



Example 1 *Independent Events*

Refer to the application above. Find the probability that it will rain on Saturday and Sunday.

$$P(A \text{ and } B) = P(A) \cdot P(B) \quad \text{Definition of independent events}$$

$$P(\text{Saturday and Sunday}) = \underbrace{P(\text{Saturday})}_{0.4} \cdot \underbrace{P(\text{Sunday})}_{0.8}$$

$$= 0.4 \cdot 0.8 \quad 40\% = 0.4 \text{ and } 80\% = 0.8$$

$$= 0.32 \quad \text{Multiply.}$$

The probability that it will rain on Saturday and Sunday is 32%.

When the outcome of one event affects the outcome of another event, the events are **dependent events**. For example, drawing a card from a deck, not returning it, then drawing a second card are dependent events because the drawing of the second card is dependent on the drawing of the first card.

Key Concept — *Probability of Dependent Events*

- **Words** If two events, *A* and *B*, are dependent, then the probability of both events occurring is the product of the probability of *A* and the probability of *B* after *A* occurs.
- **Symbols** $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

Example 2 Dependent Events

A bag contains 8 red marbles, 12 blue marbles, 9 yellow marbles, and 11 green marbles. Three marbles are randomly drawn from the bag and not replaced. Find each probability if the marbles are drawn in the order indicated.

a. *P*(red, blue, green)

The selection of the first marble affects the selection of the next marble since there is one less marble from which to choose. So, the events are dependent.

First marble: $P(\text{red}) = \frac{8}{40} \text{ or } \frac{1}{5}$ ← number of red marbles / ← total number of marbles

Second marble: $P(\text{blue}) = \frac{12}{39} \text{ or } \frac{4}{13}$ ← number of blue marbles / ← number of marbles remaining

Third marble: $P(\text{green}) = \frac{11}{38}$ ← number of green marbles / ← number of marbles remaining

$$P(\text{red, blue, green}) = P(\text{red}) \cdot P(\text{blue}) \cdot P(\text{green})$$

$$= \frac{1}{5} \cdot \frac{4}{13} \cdot \frac{11}{38} \quad \text{Substitution}$$

$$= \frac{44}{2470} \text{ or } \frac{22}{1235} \quad \text{Multiply.}$$

The probability of drawing red, blue, and green marbles is $\frac{22}{1235}$.

b. *P*(blue, yellow, yellow)

Notice that after selecting a yellow marble, not only is there one fewer marble from which to choose, there is also one fewer yellow marble.

$$P(\text{blue, yellow, yellow}) = P(\text{blue}) \cdot P(\text{yellow}) \cdot P(\text{yellow})$$

$$= \frac{12}{40} \cdot \frac{9}{39} \cdot \frac{8}{38} \quad \text{Substitution}$$

$$= \frac{864}{59{,}280} \text{ or } \frac{18}{1235} \quad \text{Multiply.}$$

The probability of drawing a blue and then two yellow marbles is $\frac{18}{1235}$.

c. *P*(red, yellow, *not* green)

Since the marble that is not green is selected after the first two marbles, there are 29 − 2 or 27 marbles that are not green.

$$P(\text{red, yellow, } not \text{ green}) = P(\text{red}) \cdot P(\text{yellow}) \cdot P(\text{not green})$$

$$= \frac{8}{40} \cdot \frac{9}{39} \cdot \frac{27}{38}$$

$$= \frac{1944}{59{,}280} \text{ or } \frac{81}{2470}$$

The probability of drawing a red, a yellow, and *not* a green marble is $\frac{81}{2470}$.

Study Tip

More Than Two Dependent Events
Notice that the formula for the probability of dependent events can be applied to more than two events.

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complement is one of
o parts that make up
whole.

In part **c** of Example 2, the events for drawing a marble that is green and for drawing a marble that is *not* green are called **complements**. Consider the probabilities for drawing the third marble.

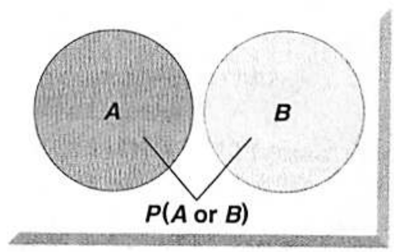
$$\underbrace{\frac{11}{38}}_{P(\text{green})} + \underbrace{\frac{27}{38}}_{P(\textit{not}\text{ green})} = \underbrace{1}_{\text{sum of probabilities}}$$

This is always true for any two complementary events.

MUTUALLY EXCLUSIVE AND INCLUSIVE EVENTS Events that cannot occur at the same time are called **mutually exclusive**. Suppose you want to find the probability of rolling a 2 *or* a 4 on a die. Since a die cannot show both a 2 and a 4 at the same time, the events are mutually exclusive.

Key Concept — *Mutually Exclusive Events*

- **Words** If two events, *A* and *B*, are mutually exclusive, then the probability that either *A* or *B* occurs is the sum of their probabilities.
- **Symbols** $P(A \text{ or } B) = P(A) + P(B)$
- **Model**



Example 3 Mutually Exclusive Events

During a magic trick, a magician randomly draws one card from a standard deck of cards. What is the probability that the card drawn is a heart or a diamond?

Since a card cannot be both a heart and a diamond, the events are mutually exclusive.

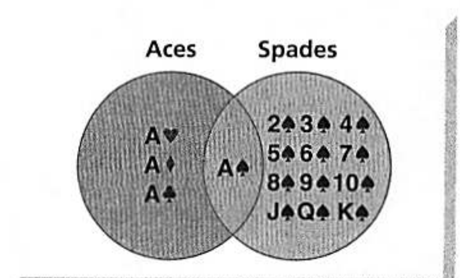
$P(\text{heart}) = \frac{13}{52}$ or $\frac{1}{4}$ ← number of hearts / ← total number of cards

$P(\text{diamond}) = \frac{13}{52}$ or $\frac{1}{4}$ ← number of diamonds / ← total number of cards

$$\begin{aligned} P(\text{heart or diamond}) &= \underbrace{P(\text{heart})} + \underbrace{P(\text{diamond})} && \text{Definition of mutually exclusive events} \\ &= \frac{1}{4} + \frac{1}{4} && \text{Substitution} \\ &= \frac{2}{4} \text{ or } \frac{1}{2} && \text{Add.} \end{aligned}$$

The probability of drawing a heart or a diamond is $\frac{1}{2}$.

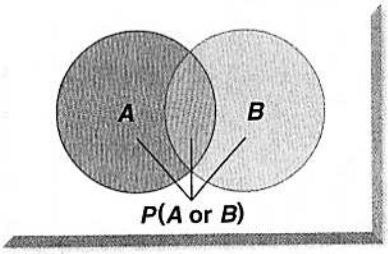
Suppose you wanted to find the probability of randomly selecting an ace or a spade from a standard deck of cards. Since it is possible to draw a card that is both an ace and a spade, these events are not mutually exclusive. They are called **inclusive** events.



If the formula for the probability of mutually exclusive events is used, the probability of drawing an ace of spades is counted twice, once for an ace and once for a spade. To correct this, you must subtract the probability of drawing the ace of spades from the sum of the individual probabilities.

Key Concept — Probability of Inclusive Events

- **Words** If two events, A and B, are inclusive, then the probability that either A or B occurs is the sum of their probabilities decreased by the probability of both occurring.
- **Model**



- **Symbols** $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Example 4 Inclusive Events

GAMES In the game of bingo, balls or tiles are numbered 1 through 75. These numbers correspond to columns on a bingo card. The numbers 1 through 15 can appear in the B column, 16 through 30 in the I column, 31 through 45 in the N column, 46 through 60 in the G column, and 61 through 75 in the O column. A number is selected at random. What is the probability that it is a multiple of 4 or is in the O column?

Since the numbers 64, 68, and 72 are multiples of 4 and they can be in the O column, these events are inclusive.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ Definition of inclusive events

$P(\text{multiple of 4 or O column})$

$$= \underbrace{P(\text{multiple of 4})}_{} + \underbrace{P(\text{O column})}_{} - \underbrace{P(\text{multiple of 4 and O column})}_{}$$

$$= \frac{18}{75} + \frac{15}{75} - \frac{3}{75} \quad \text{Substitution}$$

$$= \frac{18 + 15 - 3}{75} \quad \text{LCD is 75.}$$

$$= \frac{30}{75} \text{ or } \frac{2}{5} \quad \text{Simplify.}$$

The probability of a number being a multiple of 4 or in the O column is $\frac{2}{5}$ or 40%.

Check for Understanding

Concept Check

1. **Explain** the difference between a simple event and a compound event.
2. **Find a counterexample** for the following statement.
   If two events are independent, then the probability of both events occurring is less than 1.
3. **OPEN ENDED** Explain how dependent events are different than independent events. Give specific examples in your explanation.

4. **FIND THE ERROR** On the school debate team, 6 of the 14 girls are seniors, and 9 of the 20 boys are seniors. Chloe and Amber are both seniors on the team. Each girl calculated the probability that either a girl or a senior would randomly be selected to argue a position at a state debate.

Chloe	Amber
P(girl or senior)	P(girl or senior)
$= \frac{14}{34} + \frac{15}{34} - \frac{6}{34}$	$= \frac{6}{34} + \frac{15}{34} - \frac{14}{34}$
$= \frac{23}{34}$	$= \frac{7}{34}$

Who is correct? Explain your reasoning.

Guided Practice

A bin contains 8 blue chips, 5 red chips, 6 green chips, and 2 yellow chips. Find each probability.

5. drawing a red chip, replacing it, then drawing a green chip
6. selecting two yellow chips without replacement
7. choosing green, then blue, then red, replacing each chip after it is drawn
8. choosing green, then blue, then red without replacing each chip

A student is selected at random from a group of 12 male and 12 female students. There are 3 male students and 3 female students from each of the 9th, 10th, 11th, and 12th grades. Find each probability.

9. *P*(9th or 12th grader)
10. *P*(10th grader or female)
11. *P*(male or female)
12. *P*(male or not 11th grader)

Application

BUSINESS For Exercises 13–15, use the following information.
Mr. Salyer is a buyer for an electronics store. He received a shipment of 5 DVD players in which one is defective. He randomly chose 3 of the DVD players to test.

13. Determine whether choosing one DVD player after another indicates independent or dependent events.
14. What is the probability that he selected the defective player?
15. Suppose the defective player is one of the three that Mr. Salyer tested. What is the probability that the last one tested was the defective one?

Practice and Apply

Homework Help

For Exercises	See Examples
16–19, 24, 25, 28–31	2
20–23, 32–34	1
26, 27, 41, 44, 45	4
36–40, 42, 43, 46, 47	3

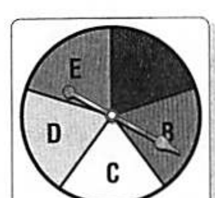
Extra Practice

See page 851.

A bag contains 2 red, 6 blue, 7 yellow, and 3 orange marbles. Once a marble is selected, it is not replaced. Find each probability.

16. *P*(2 orange)
17. *P*(blue, then red)
18. *P*(2 yellows in a row then orange)
19. *P*(blue, then yellow, then red)

A die is rolled and a spinner like the one at the right is spun. Find each probability.



20. *P*(3 and D)
21. *P*(an odd number and a vowel)
22. *P*(a prime number and A)
23. *P*(2 and A, B, or C)

www.algebra1.com/self_check_quiz

Raffle tickets numbered 1 through 30 are placed in a box. Tickets for a second raffle numbered 21 to 48 are placed in another box. One ticket is randomly drawn from each box. Find each probability.

24. Both tickets are even.

25. Both tickets are greater than 20 and less than 30.

26. The first ticket is greater than 10, and the second ticket is less than 40 or odd.

27. The first ticket is greater than 12 or prime, and the second ticket is a multiple of 6 or a multiple of 4.

More About. . .

Safety

In the U.S., 60% of carbon monoxide emissions come from transportation sources. The largest contributor is highway motor vehicles. In urban areas, motor vehicles can contribute more than 90%.

Source: U.S. Environmental Protection Agency

SAFETY For Exercises 28–31, use the following information.
A carbon monoxide detector system uses two sensors, A and B. If carbon monoxide is present, there is a 96% chance that sensor A will detect it, a 92% chance that sensor B will detect it, and a 90% chance that both sensors will detect it.

28. Draw a Venn diagram that illustrates this situation.

29. If carbon monoxide is present, what is the probability that it will be detected?

30. What is the probability that carbon monoxide would go undetected?

31. Do sensors A and B operate independently of each other? Explain.

BIOLOGY For Exercises 32–34, use the table and following information.
Each person carries two types of genes for eye color. The gene for brown eyes (B) is dominant over the gene for blue eyes (b). That is, if a person has one gene for brown eyes and the other for blue, that person will have brown eyes. The Punnett square at the right shows the genes for two parents.

	B	b
B	BB	Bb
b	Bb	bb

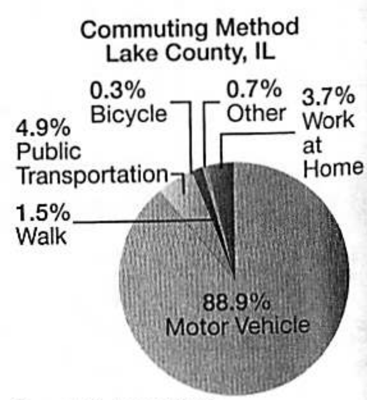
32. What is the probability that any child will have blue eyes?

33. What is the probability that the couple's two children both have brown eyes?

34. Find the probability that the first or the second child has blue eyes.

35. **RESEARCH** Use the Internet or other reference to investigate various blood types. Use this information to determine the probability of a child having blood type O if the father has blood type A(Ai) and the mother has blood type B(Bi).

TRANSPORTATION For Exercises 36 and 37, use the graph and the following information.
The U.S. Census Bureau conducted an American Community Survey in Lake County, Illinois. The circle graph at the right shows the survey results of how people commute to work.



Source: U.S. Census Bureau

36. If a person from Lake County was chosen at random, what is the probability that he or she uses public transportation or walks to work?

37. If offices are being built in Lake County to accommodate 400 employees, what is the minimum number of parking spaces an architect should plan for the parking lot?

ECONOMICS **For Exercises 38–40, use the table below that compares the total number of hourly workers who earned the minimum wage of $5.15 with those making less than minimum wage.**

Number of Hourly Workers (thousands)			
Age (years)	Total	At $5.15	Below $5.15
16–24	15,793	1145	2080
25+	55,287	970	2043

Source: U.S. Bureau of Labor Statistics

38. If an hourly worker was chosen at random, what is the probability that he or she earned minimum wage? less than minimum wage?

39. What is the probability that a randomly-chosen hourly worker earned less than or equal to minimum wage?

40. If you randomly chose an hourly worker from each age group, which would you expect to have earned no more than minimum wage? Explain.

More About. . .

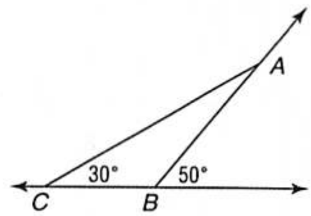
Economics

The first federal minimum wage was set in 1938 at $0.25 per hour. That was the equivalent of $3.05 in 2000.

Source: U.S. Department of Labor

GEOMETRY **For Exercises 41–43, use the figure and the following information.**

Two of the six non-straight angles in the figure are chosen at random.

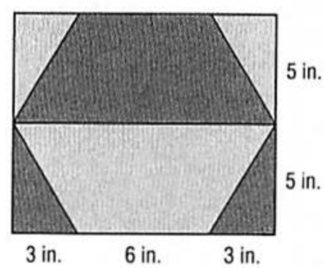


41. What is the probability of choosing an angle inside $\triangle ABC$ or an obtuse angle?

42. What is the probability of selecting a straight angle or a right angle inside $\triangle ABC$?

43. Find the probability of picking a 20° angle or a 130° angle.

A dart is thrown at a dartboard like the one at the right. If the dart can land anywhere on the board, find the probability that it lands in each of the following.



44. a triangle or a red region

45. a trapezoid or a blue region

46. a blue triangle or a red triangle

47. a square or a hexagon

CRITICAL THINKING **For Exercises 48–51, use the following information.**

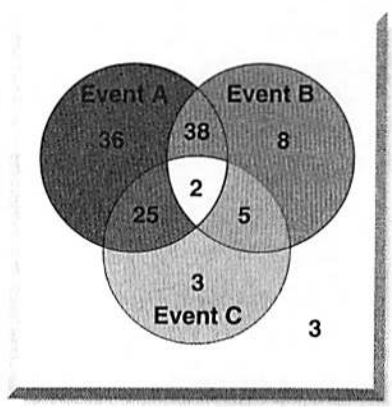
A sample of high school students were asked if they:

A) drive a car to school,

B) are involved in after-school activities, or

C) have a part-time job.

The results of the survey are shown in the Venn diagram.



48. How many students were surveyed?

49. How many students said that they drive a car to school?

50. If a high school student is chosen at random, what is the probability that he or she does all three?

51. What is the probability that a randomly-chosen student drives a car to school or is involved in after-school activities or has a part-time job?

52. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are probabilities used by meteorologists?

Include the following in your answer:

- a few sentences about how compound probabilities can be used to predict the weather, and
- assuming that the events are independent, the probability that it will rain either Saturday or Sunday if there is a 30% chance of rain on Saturday and a 50% chance of rain Sunday.

Standardized Test Practice

53. A bag contains 8 red marbles, 5 blue marbles, 4 green marbles, and 7 yellow marbles. Five marbles are randomly drawn from the bag and not replaced. What is the probability that the first three marbles drawn are red?

(A) $\frac{1}{27}$ (B) $\frac{28}{1771}$ (C) $\frac{7}{253}$ (D) $\frac{7}{288}$

54. Yolanda usually makes 80% of her free throws. What is the probability that she will make at least one free throw in her next three attempts?

(A) 99.2% (B) 51.2% (C) 38.4% (D) 9.6%

Maintain Your Skills

Mixed Review

CIVICS For Exercises 55 and 56, use the following information.
The Stratford town council wants to form a 3-person parks committee. Five people have applied to be on the committee. *(Lesson 14-2)*

55. How many committees are possible?

56. What is the probability of any one person being selected if each has an equal chance?

57. **BUSINESS** A real estate developer built a strip mall with seven different-sized stores. Ten small businesses have shown interest in renting space in the mall. The developer must decide which business would be best suited for each store. How many different arrangements are possible? *(Lesson 14-1)*

Find each sum or difference. *(Lesson 13-2)*

58. $\begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 5 \end{bmatrix}$

59. $\begin{bmatrix} -4 & -5 \\ 8 & 8 \end{bmatrix} - \begin{bmatrix} -9 & -7 \\ 4 & 9 \end{bmatrix}$

60. Find the quotient of $\frac{2m^2 + 7m - 15}{m + 5}$ and $\frac{9m^2 - 4}{3m + 2}$. *(Lesson 12-4)*

Simplify. *(Lesson 11-1)*

61. $\sqrt{45}$

62. $\sqrt{128}$

63. $\sqrt{40b^4}$

64. $\sqrt{120a^3b}$

65. $3\sqrt{7} \cdot 6\sqrt{2}$

66. $\sqrt{3}(\sqrt{3} + \sqrt{6})$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Express each fraction as a decimal. Round to the nearest thousandth. *(To review **expressing fractions as decimals**, see pages 804 and 805.)*

67. $\frac{9}{24}$

68. $\frac{2}{15}$

69. $\frac{63}{128}$

70. $\frac{5}{52}$

71. $\frac{8}{36}$

72. $\frac{11}{38}$

73. $\frac{81}{2470}$

74. $\frac{18}{1235}$

75. $\frac{128}{3570}$