

14-4 Probability Distributions

What You'll Learn

- Use random variables to compute probability.
- Use probability distributions to solve real-world problems.

How can a pet store owner use a probability distribution?

The owner of a pet store asked customers how many pets they owned. The results of this survey are shown in the table.

Number of Pets	Number of Customers
0	3
1	37
2	33
3	18
4	9



Vocabulary

- random variable
- probability distribution
- probability histogram

RANDOM VARIABLES AND PROBABILITY A **random variable** is a variable whose value is the numerical outcome of a random event. In the situation above, we can let the random variable X represent the number of pets owned. Thus, X can equal 0, 1, 2, 3, or 4.

Example 1 Random Variable

Refer to the application above.

- a. Find the probability that a randomly-chosen customer has 2 pets.

There is only one outcome in which there are 2 pets owned, and there are 100 survey results.

$$\begin{aligned} P(X = 2) &= \frac{2 \text{ pets owned}}{\text{customers surveyed}} \\ &= \frac{33}{100} \end{aligned}$$

The probability that a randomly-chosen customer has 2 pets is $\frac{33}{100}$ or 33%.

- b. Find the probability that a randomly-chosen customer has at least 3 pets.

There are $18 + 9$ or 27 outcomes in which a customer owns at least 3 pets.

$$P(X \geq 3) = \frac{27}{100}$$

The probability that a randomly-chosen customer owns at least 3 pets is $\frac{27}{100}$ or 27%.

Study Tip

Reading Math

The notation $P(X = 2)$ means the same as $P(2 \text{ pets})$, the probability of a customer having 2 pets.

PROBABILITY DISTRIBUTIONS The probability of every possible value of the random variable X is called a **probability distribution**.

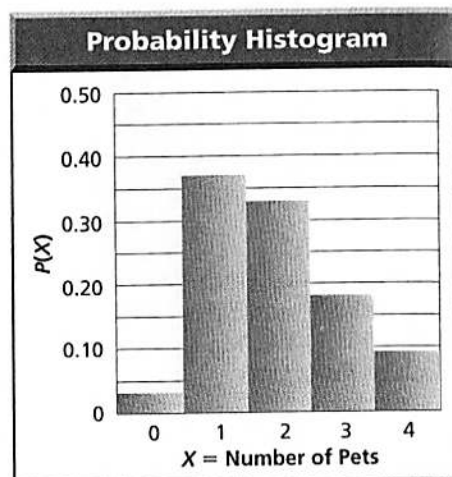
Key Concept

Properties of Probability Distributions

1. The probability of each value of X is greater than or equal to 0 and less than or equal to 1.
2. The probabilities of all of the values of X add up to 1.

The probability distribution for a random variable can be given in a table or in a **probability histogram**. The probability distribution and a probability histogram for the application at the beginning of the lesson are shown below.

Probability Distribution Table	
$X = \text{Number of Pets}$	$P(X)$
0	0.03
1	0.37
2	0.33
3	0.18
4	0.09



Example 2 Probability Distribution

CARS The table shows the probability distribution of the number of vehicles per household for the Columbus, Ohio, area.

a. Show that the distribution is valid.

Check to see that each property holds.

- For each value of X , the probability is greater than or equal to 0 and less than or equal to 1.
- $0.10 + 0.42 + 0.36 + 0.12 = 1$, so the probabilities add up to 1.

Vehicles per Household Columbus, OH	
$X = \text{Number of Vehicles}$	Probability
0	0.10
1	0.42
2	0.36
3+	0.12

Source: U.S. Census Bureau

b. What is the probability that a household has fewer than 2 vehicles?

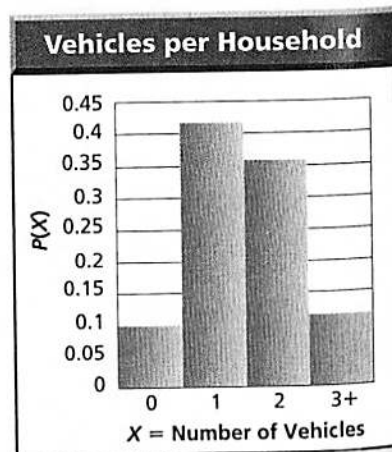
Recall that the probability of a compound event is the sum of the probabilities of each individual event.

The probability of a household having fewer than 2 vehicles is the sum of the probability of 0 vehicles and the probability of 1 vehicle.

$$\begin{aligned}
 P(X < 2) &= P(X = 0) + P(X = 1) && \text{Sum of individual probabilities} \\
 &= 0.10 + 0.42 \text{ or } 0.52 && P(X = 0) = 0.10, P(X = 1) = 0.42
 \end{aligned}$$

c. Make a probability histogram of the data.

Draw and label the vertical and horizontal axes. Remember to use equal intervals on each axis. Include a title.



Cars

In 1900, there were 8000 registered cars in the United States. By 1998, there were over 131 million registered cars. This is an increase of more than 1,637,400%.

Source: *The World Almanac*

Check for Understanding

Concept Check

- List the conditions that must be satisfied to have a valid probability distribution.
- Explain why the probability of tossing a coin three times and getting 1 head and 2 tails is the same as the probability of getting 1 tail and 2 heads.
- OPEN ENDED** Describe a situation that could be displayed in a probability histogram.

Guided Practice

For Exercises 4–6, use the table that shows the possible sums when rolling two dice and the number of ways each sum can be found.

Sum of Two Dice	2	3	4	5	6	7	8	9	10	11	12
Ways to Achieve Sum	1	2	3	4	5	6	5	4	3	2	1

- Draw a table to show the sample space of all possible outcomes.
- Find the probabilities for $X = 4$, $X = 5$, and $X = 6$.
- What is the probability that the sum of two dice is greater than 6 on each of three separate rolls?

Application

GRADES For Exercises 7–9, use the table that shows a class's grade distribution, where A = 4.0, B = 3.0, C = 2.0, D = 1.0, and F = 0.

X = Grade	0	1.0	2.0	3.0	4.0
Probability	0.05	0.10	0.40	0.40	0.05

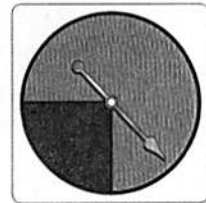
- Show that the probability distribution is valid.
- What is the probability that a student passes the course?
- What is the probability that a student chosen at random from the class receives a grade of B or better?

Practice and Apply

Homework Help

For Exercises	See Examples
10, 11, 14, 18	1
12, 13, 15–17, 19–22	2

For Exercises 10–13, the spinner shown is spun three times.



- Write the sample space with all possible outcomes.
- Find the probability distribution X , where X represents the number of times the spinner lands on blue for $X = 0$, $X = 1$, $X = 2$, and $X = 3$.
- Make a probability histogram.
- Do all possible outcomes have an equal chance of occurring? Explain.

Extra Practice

See page 852.

SALES For Exercises 14–17, use the following information.

A music store manager takes an inventory of the top 10 CDs sold each week. After several weeks, the manager has enough information to estimate sales and make a probability distribution table.

Number of Top 10 CDs Sold Each Week	0–100	101–200	201–300	301–400	401–500
Probability	0.10	0.15	0.40	0.25	0.10

- Define a random variable and list its values.
- Show that this is a valid probability distribution.
- In a given week, what is the probability that no more than 400 CDs sell?
- In a given week, what is the probability that more than 200 CDs sell?



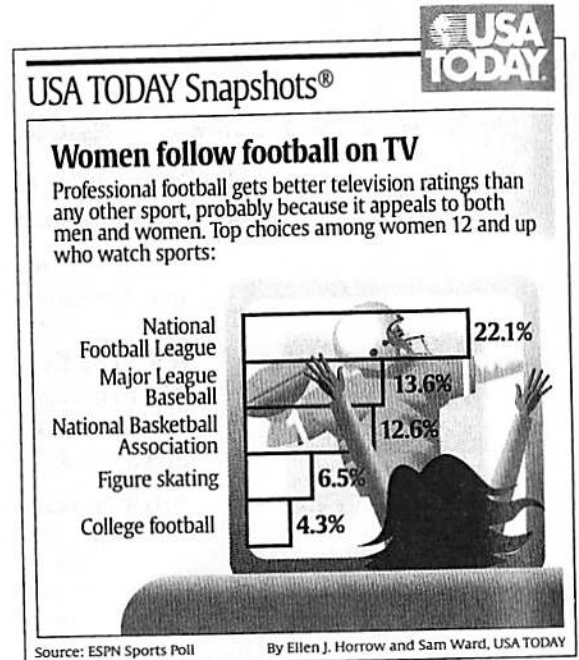
EDUCATION For Exercises 18–20, use the table that shows the education level of persons aged 25 and older in the United States.

X = Level of Education	Probability
Some High School	0.167
High School Graduate	0.333
Some College	0.173
Associate's Degree	0.075
Bachelor's Degree	0.170
Advanced Degree	0.082

Source: U.S. Census Bureau

- If a person was randomly selected, what is the probability that he or she completed at most some college?
- Make a probability histogram of the data.
- Explain how you can find the probability that a randomly selected person has earned at least a bachelor's degree.

SPORTS For Exercises 21 and 22, use the graph that shows the sports most watched by women on TV.



- Determine whether this is a valid probability distribution. Justify your answer.
- Based on the graph, in a group of 35 women how many would you expect to say they watch figure skating?
- CRITICAL THINKING** Suppose a married couple has children until they have a girl. Let the random variable X represent the number of children in their family.
 - Calculate the probabilities for $X = 1, 2, 3,$ and 4 .
 - Find the probability that the couple will have more than 4 children.
- WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can a pet store owner use a probability distribution?

Include the following in your answer:

- a sentence or two describing how to create a probability distribution, and
- an explanation of how the store owner could use a probability distribution to establish a frequent buyer program.



- The table shows the probability distribution for the number of heads when four coins are tossed. What is the probability that there are no more than two heads showing on a random toss?

X = Number of Heads	0	1	2	3	4
Probability $P(X)$	0.0625	0.25	0.375	0.25	0.0625

- (A) 0.6875 (B) 0.375 (C) 0.875 (D) 0.3125
- On a random roll of two dice, what is the probability that the sum of the numbers showing is less than 5?

(A) 0.08 (B) 0.17 (C) 0.11 (D) 0.28

Maintain Your Skills

Mixed Review A card is drawn from a standard deck of 52 cards. Find each probability.

(Lesson 14-3)

27. $P(\text{ace or } 10)$ 28. $P(3 \text{ or diamond})$ 29. $P(\text{odd number or spade})$

Evaluate. (Lesson 14-2)

30. ${}_{10}C_7$ 31. ${}_{12}C_5$ 32. $({}_6P_3)({}_5P_3)$

Let $A = \begin{bmatrix} 1 & 4 \\ 5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 0 \\ -2 & 5 \end{bmatrix}$. (Lesson 13-2)

33. Find $A + B$. 34. Find $B - A$.

Write an inverse variation equation that relates x and y . Assume that y varies inversely as x . Then solve. (Lesson 12-1)

35. If $y = -2.4$ when $x = -0.6$, find y when $x = 1.8$.
36. If $y = 4$ when $x = -1$, find x when $y = -3$.

Simplify each expression. (Lesson 11-2)

37. $3\sqrt{8} + 7\sqrt{2}$ 38. $2\sqrt{3} + \sqrt{12}$ 39. $3\sqrt{7} - 2\sqrt{28}$

SAVINGS For Exercises 40–42, use the following information.

Selena is investing her \$900 tax refund in a certificate of deposit that matures in 4 years. The interest rate is 8.25% compounded quarterly. (Lesson 10-6)

40. Determine the balance in the account after 4 years.
41. Her friend Monique invests the same amount of money at the same interest rate, but her bank compounds interest monthly. Determine how much she will have after 4 years.
42. Which type of compounding appears more profitable? Explain.

Getting Ready for the Next Lesson

PREREQUISITE SKILL Write each fraction as a percent rounded to the nearest whole number. (To review writing fractions as percents, see pages 804 and 805.)

43. $\frac{16}{80}$ 44. $\frac{20}{52}$ 45. $\frac{30}{114}$
46. $\frac{57}{120}$ 47. $\frac{72}{340}$ 48. $\frac{54}{162}$

Practice Quiz 2

Lessons 14-3 and 14-4

For Exercises 1–3, use the probability distribution for the number of people in a household. (Lesson 14-4)

- Show that the probability distribution is valid.
- If a household is chosen at random, what is the probability that 4 or more people live in it?
- Make a histogram of the data.

A ten-sided die, numbered 1 through 10, is rolled. Find each probability.

- $P(\text{odd or greater than } 4)$
- $P(\text{less than } 3 \text{ or greater than } 7)$

American Households	
$X = \text{Number of People}$	Probability
1	0.25
2	0.32
3	0.18
4	0.15
5	0.07
6	0.02
7+	0.01

Source: U.S. Census Bureau